Preliminary Draft – Comments Welcome

Price-Concentration Analysis: Ending the Myth, and Moving Forward

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Abstract

Price-concentration studies have a long history empirical industrial organization, including prominent recent examples. Although their robustness has been questioned for many years, a casual review of the literature leaves the impression that the main issue is the endogeneity of the concentration measure, and that once this is addressed, the problem is solved. This perspective ignores the bigger issue: the equations estimated in most price-concentration studies lack an economic foundation. Critiques in Economic Handbooks allude to the foundational issue, but the IO literature has not explored it in depth or discussed the implications for interpreting empirical results. This paper boils the problem down to the core issue, tackles interpretation, and discusses the way forward.

Keywords: Price, concentration, structure conduct performance, mergers, partial ownership, free entry equilibrium

JEL Classifications: D43, G23, G34, K21, L13, L41

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— Concentration is an effect, not a cause. Prices in markets depend on demand, production possibilities, and ownership and control. So does concentration.

I. Introduction and Overview

Price-concentration studies have a long history in industrial organization, and they remain a prominent component of the empirical IO tool kit.[2] However, this branch of IO is clearly a source of trepidation. The strongest criticism appears in Economic Handbook chapters that tend to emphasize economic foundations.[3] Whinston (2007) summarizes economists' uneasiness about these studies in his chapter on antitrust policy toward horizontal mergers:

> "Given the relative ease and widespread use of this method, one might hope that it gives at least approximately correct answers despite these problems. It would be good to know more than we now do about whether this is right."[4]

In this paper I present an impossibility result and related analysis that imply there is no reason to expect price-concentration studies to give correct answers. I show that in canonical oligopoly models, the functional relationship between price and concentration posited in most of this literature does not exist. I believe this lies at the core of economists' uneasiness with this method. I also explain the implications of economic theory for the interpretation of empirical estimates from price-concentration studies the literature. The interpretation implied by theory is radically different than the interpretation generally given in the literature.

Anticipating several immediate questions and objections, let me break with tradition for introductions and inject just enough mathematical structure to identify precisely the core issue. The main ideas in the paper are identified here in the introduction, and specific details, examples, and discussion are provided in the rest of the paper.

Consider a scenario in which one firm in some market acquires a financial interest C in another competitor. We can think of $C = 0$ as the status quo, $C = 1$ as a complete merger, and intermediate values of C as partial ownership interests that create some degree of "common ownership" in the market.[5] For this scenario, assume that C is exogenous, so that the potential endogeneity of the mergers and acquisitions process is not a concern. In merger policy, we are often interested in how

[2] Evans et al. (1994) observe that nearly a hundred such studies were conducted before Bresnahan's (1989) IO Handbook chapter introducing the "new empirical industrial organization" (NEIO). Since then, researchers have continued to apply this technique to a wide range of industries, including airlines (Borenstein, 1989; Kim and Singal, 1993; Evans et al. (1993); Azar et al., 2017); bank deposits (Prager and Hannan, 1998; Focarelli and Panetta, 2003; Azar et al. (2016)); movie theaters (Davis, 2005); office supply superstores (Manuszak and Moul, 2008); gasoline (Zimmerman, 2012); and health insurance (Dafny et al., 2012; Trish and Herring, 2015); and beer (Ashenfelter et al., 2015). Others use concentration to explain compensation (e.g. Aggarwal and Samwick, 1999; Anton et al., 2016; Kwon, 2016)

[3] Bresnahan (1989), pp. 1042-1044; Whinston (2007), pp. 2411-2014; Reiss and Wolak (2007), pp. 4300-4301.

[4] Whinston (2007), p. 2414.

[5] By "common ownership" I mean a situation where one or more owners hold shares in more than one competitor in the market. A formal framework for modeling the competitive effects of mergers and partial acquisitions that create common ownership is presented in the next section.

much an increase in C will raise equilibrium prices. What might we learn about this question by estimating a relationship between price and concentration?

Economic theory tells us that equilibrium prices and quantities depend on the common ownership variable C and all other exogenous factors X that influence the market. Write the equilibrium price and quantity vectors as $P(C, X)$ and $Q(C, X)$, the "reduced-form" representations of price and quantity familiar from econometrics textbooks. Concentration is generally defined as a function of quantities or revenues, and because these variables depend on C and X, equilibrium concentration also depends on C and X. Write equilibrium concentration as $H(C, X)$, another reduced-form. We can now see at a high level of generality (no restrictions on the cost and demand functions or the nature of the oligopoly game) what economic theory tells us about the price-concentration relationship. Theory generates *parametric equations* that define a curve in price-concentration space.[6] As the fractional ownership interest C and other factors X change, price and concentration vary parametrically with each other according to the functions $P(C, X)$ and $H(C, X)$.

In the price-concentration literature, researchers typically posit a direct *functional* relationship between price and concentration (and other factors X), not a parametric one.[7] A threshold question is whether economic reasoning yields this functional relationship. The existence of such a relationship in theory is a basic requirement for empirical estimates of the relationship to have a clear economic interpretation.

A parametric relationship between two variables does not by itself rule out the existence of a functional relationship. For example, if it were possible to invert equilibrium concentration $H(C, X)$ with respect to C to obtain $C = g(H, X)$, then it would be possible to write price as a function of concentration and exogenous factors as $p(H, X) = P\big(g(H, X), X\big)$. One could then obtain the relationship between price and the merger by estimating $p(H, X)$ and $H(C, X)$ and studying $P(H(C, X), X)$.[8] Unfortunately, $H(C, X)$ is *not* invertible over relevant domains under standard definitions of concentration in standard economic environments. This is the core issue. In general, economic theory does not imply the existence of a function in which price depends on concentration and other factors over the relevant economic domain, i.e., over a domain that includes valid functions that define the economic environment (e.g., supply and demand relations) and feasible values of the variables in that environment.

Some price-concentration studies relate the change in price to the change in concentration that arises from the change in ownership and other factors. Economic theory does not generally yield this relationship either.

The invertibility problem arises because conditional on X, it generally is not possible to associate a particular acquisition C with each level of concentration H. For single-dimensional acquisitions as

[6] In mathematics, parametric equations of a curve express the coordinates of the points of the curve as functions of one or more variables referred to as "parameters." In this context, the parameters are C and X. These are not to be confused with parameters in an econometric specification that treats C and X as explanatory variables.

[7] By "functional relationship" I mean that each value of (C, X) maps into exactly one value of price. This is the standard definition of a function.

[8] Why one might want to do this rather than estimate $P(C, X)$ directly is an obvious question taken up later. The point here is simply to understand how to interpret the price-concentration relationship.

in the example here, the problem is that concentration H generally is not a monotonic function of C. This means that a given concentration level may may arise from two or more different acquisitions that generate two or more different prices. The problem is even more apparent when the acquisition C has higher dimension than H, as when C is a matrix reflecting the ownership interests of one or more owners and the control they have over the firms they own.[9] The concentration index often used in this case is the modified-HHI (MHHI), which allows partial ownership interests by multiple owners and different assumptions about how ownership translates into control.[10] In general, many different ownership scenarios generate a given MHHI. Again, the concentration-acquisition relationship is not invertible, and there is no functional relationship between equilibrium prices and concentration as measured by the MHHI over relevant domains.

The invertibility problem creates obvious difficulties interpreting results from price-concentration regressions in the context of merger analysis. Acquisitions that increase price may increase or decrease concentration; acquisitions that increase concentration may increase or decrease price. This means that in data, econometric estimates of the relationship between price and the MHHI need not say anything about the relationship between price and the relevant ownerhship/control variable C. In econometric language, price-concentration regressions do not identify economic coefficients of interest, which in the merger context are those that inform the relationship between price and the acquisition. Although the potential for endogenous entry and acquisitions may further obscure the interpretation of coefficients from such regressions, the core issue is the invertibility problem, not "endogeneity" as this term is typically used in econometrics.[11]

More formally, the relationship $p(H, X)$ is neither a structural nor a reduced form relationship implied by an economic model. In standard oligopoly models, the function does not exist except over limited domains. Over domains in which $p(H, X)$ exists, both the sign and magnitude of the relationship between price and concentration depend on the details of the oligopoly model. It is not possible to infer even the sign of the relationship between price and the acquisition C from the relationship $p(H, X)$ without knowing both the details of the oligopoly model and the specific restrictions on the domain of variables.

The reader might object that it is widely accepted that there *is* a relationship between price and the HHI (or MHHI) under two theories of oligopoly—that of Cournot[12], and that of Stigler (1964). However, as Whinston (2007) observes, the Cournot equilibrium price is a function of the HHI and all exogenous factors only in the special case of symmetric, homogenous firms with constant marginal cost. More generally, Cournot's model can be shown to yield a supply relation of the form $P = f(Q, H, X^s)$ where Q is the vector of competing firms' quantities, H is the modified-HHI (allowing for partial ownership, with the HHI as a special case), X^s includes the components of X

[9]The theory of partial ownership under which C represents the effects of ownership and control on incentives is described later.

[10]O'Brien and Salop (2000).

[11]The invertibility problem is related to endogeneity in that it arises because concentration is endogenous. However, the use of systems estimations techniques to treat the endogeneity of concentration does not solve the invertibility problem and the associated interpretation problem, as I explain below.

[12]Cowling and Waterson (1976) discuss this relationship.

that shift the supply relations but not demand, and the function f takes a particular form implied by the theory. One can also interpret Stigler's theory of oligopoly using this functional relationship by embedding a conduct parameter in f that depends on concentration. In principle, one can estimate this "structural" equation given enough instruments for Q and H. Indeed, this would amount to estimating a structural supply relation along the lines of that discussed in Bresnahan (1989). However, this is not what price-concentration studies do.

Most price concentration studies estimate equations that take one of two forms in the context of the simplified model discussed in this introduction:

$$P = f(H, X; \theta, \epsilon), \tag{1}$$

$$P = f(H, C, X; \theta, \epsilon). \tag{2}$$

where θ is a vector of parameters, ϵ is an econometric error, and the price and concentration variables may be in levels or changes. In light of the observations in the preceding paragraphs, what is interpretation of the estimated coefficients in these relationships?

Equation (2) is the easiest case because economic theory implies that C and X are sufficient to determine P, given θ and ϵ. If the theory is correct, then the inclusion of H in the equation amounts to adding an extraneous variable. Suppose the relationship in (2) is linear in the parameters with an additive, mean-zero statistical error, and that the researcher uses standard regression techniques to estimate it. If H is uncorrelated with the error (e.g., if the researcher uses valid instrumental variables to replace H with some $\hat{H}$ from a first stage regression), then the inclusion of H (specifically $\hat{H}$) simply adds noise. The researcher should find that price does not depend on H at all, as this is the prediction of the theory. A different finding would mean that the theory is wrong, the equation is mis-specified, or both. If H is correlated with the error, then the coefficients on C and X yield biased estimates of the true parameters in the reduced-form relationship. The coefficient on H will depend on the correlations between H, the error, and the other explanatory variables. By itself, the coefficient has no theoretical meaning beyond the variation in P that it absorbs due to mis-specification.

Equation (1) amounts to using H (or $\hat{H}$ if instrumental variables are employed) as a proxy for C in a reduced-form relationship. The issues are no different than the use of proxy variables generally. In a linear regression, if the difference between H and C is uncorrelated with the error, then H is a valid proxy, and the greater is the correlation between H and C, the better the proxy. However, H is likely to be a poor proxy for C. One reason is the invertibility problem — a one-to-one mapping from the acquisition variable C to concentration generally does not exist. A second reason is that even if the domain is restricted in a way that generates a one-to-one mapping from C to H, the relationship in the data is likely to be noisy.

Observe that the use of instrumental variables to deal with the endogeneity of H does not solve the problem raised here. No amount of econometric sophistication in the estimation of (1) or (2) will do so. The issue is not endogenity *per se*, but that economic theory provides no foundation for these relationships. Thus, estimates of these relationships have no obvious economic interpretation.

Related Literature and Contribution of the Paper. Concerns about how to interpret studies that use concentration as an explanatory variable are very old, tracing to at least Bain (1951). In the remainder of this section, I offer a brief historical sketch that puts the issues in context and highlights what this paper contributes.

Early work in empirical IO explored the relationship between profit and concentration. In a pioneering study, Bain (1951) found that firms in highly concentrated industries had higher profit rates on average than firms in industries of lower concentration.[13] Although he is generally credited with originating the oft-criticised structure-concentration-performance paradigm that was the cornerstone of IO into the 1970s, an interesting fact is that he himself recognized that his findings did not illuminate the relationship between concentration and competitive performance:

> "But the existence of a low profit rate may be associated with adverse results on other levels (such as chronic excess capacity) and any profit performance must be read in the light of the rate of technical progress, the trend of demand, and so forth. We are thus essentially unable to discover any net relation of concentration to the workability of competition; we seek simply the relation of concentration to the profit rate, whatever its ultimate significance."[14]

A plausible reading of Bain's cautionary statement is that he understood that the economic interpretation of any relationship between profit and concentration would require an economic model of the factors affecting these variables.

Demsetz (1973) was more explicit in raising concerns similar to those raised by Bain. He pointed out that high profits can be explained just as easily by low costs as by anticompetitive behavior. Because efficient firms tend to have high shares, industries with a small number of highly efficient firms tend to be highly concentrated and exhibit high profitability. However, such industries may perform better (e.g., generate lower prices or higher welfare) than they would if the low cost firms had higher costs and the market were less concentrated. In other words, the profit-concentration relationship by itself is not informative about the relationship between concentration and competitive performance.

The concerns raised by Bain and Demsetz do not necessarily imply that concentration is a poor predictor of price or some other measure of performance, as their studies do not address whether concentration would be a good predictor holding other factors equal (e.g., excess capacity, variable costs, other factors). Farrell and Shapiro (1990a, 1990b) conducted the first formal theoretical analysis of the role of concentration in assessing competitive performance, focusing on its use in merger analysis. They showed that under Counot oligopoly, increases in equilibrium market concentration (as measured by the HHI) are often positively associated with improvements in welfare (as measured by total surplus). This analysis raised a serious red flag for the prospect of drawing strong inferences about changes in market performance from changes in concentration,

[13] Bain (1951), p. 323.
[14] Bain (1951), p. 294.

including changes induced by merger. Indeed, Farrell and Shapiro state: "Although our analysis reveals that conventional policy, which aims to avert increases in measured concentration, lacks an explicit theoretical foundation, our inquiry has as yet furnished no clear alternative."[15]

One of the first researchers to raise specific concerns about using concentration as an explanatory variable for price in empirical work was Bresnahan (1989) in his IO Handbook chapter that introduced the "new empirical industrial organization" (NEIO). In his discussion of this technique, Bresnahan stated:

> "Most of these [price-concentration] studies offer the interpretation that the empirically estimated relationship can be interpreted to cast light the prediction of almost all theories of oligopoly that higher concentration causes higher price-cost margins by changing conduct. I have seen no careful defense of this interpretation, and I am troubled by it; I offer a series of interpretational difficulties here not because I believe they are true but because they have not yet been rebutted."

Evans et al. (1993) interpreted Bresnahan's main concern to be the endogeneity of the concentration measure and showed how this can be addressed using instrumental variables techniques. Much of the empirical literature heeds this concern and attempts to address it. However, the issues raised by Bresnahan go beyond endogeneity in the empirical specification and are not separable from the theoretical issues raised in this paper.

Bresnahan made three main points about price-concentration methodology. First, in reference to environments in which entry responds endogenously to market size, Bresnahan asked: "what relationship [between price and concentration] are we seeing in the data?" (p. 1043). To interpret this question in the context of the simplified analytical scenario presented above, imagine that the common ownership variable C is fixed in the data, but that there is an endogenous entry process that also depends on X, which now measures market size. The equilibrium values of price, quantity, and entry all depend on market size, and because the concentration measure depends on price, quantity, and entry, equilibrium concentration also depends on market size. The relationship between price and concentration is once again parametric. Whether a functional relationship between price and concentration exists depends on whether concentration is monotonic in market size. Perhaps the relationship is monotonic, but perhaps not. For example, suppose that for market sizes below a certain threshold, higher market size attracts new entrants with costs similar to incumbents. Over this range of market sizes, we expect entry to reduce concentration and reduce price. On the other hand, suppose that a potential entrant with lower variable costs than incumbents enter only if market size exceeds some threshold. If the new entrant takes sufficient share from incumbents, then concentration could rise, even as price falls. If one simply studies the relationship between price and concentration across markets or over time, the relationship would not accurately capture effect of a merger or collusion among firms because it would also reflect a change in industry cost

[15]Farrell and Shapiro (1990b), p. 290. Farrel and Shapiro establish several insights about the effects of mergers between Cournot competitors, building on the work of Salant et al. (1983) and Perry and Porter (1985). Among other results, they show that profitable Cournot mergers raise price unless the merger generates sufficient synergies, and that the welfare effects of mergers depend on firms' shares and the responsiveness of firms' quantities to changes in their rivals' quantities.

structure. The issue here, like the common ownership example, is one of interpretation—what does the relationship between price and concentration mean? The answer cannot be found by using econometric techniques that deal with the endogeneity of concentration.

The second issue raised by Bresnahan applies even when the number of competitors is fixed. Other factors equal, a higher degree of cost heterogeneity in a market tends to yield higher concentration, but "greater heterogeneity in costs might interact with conduct in a way that increases prices, or it may not." (p. 1044). As one illustration in the context of the simplified analytical scenario presented earlier, imagine that the common ownership variable C does not vary in the data and that X includes firms' marginal costs, which happen to vary in the data over time or across markets about a fixed mean (just as an example). Under Cournot competition (for illustration), P would not vary with X in this market. (The Cournot equilibrium price is a function of the unweighted average marginal cost in the market). However, concentration would vary with cost heterogeneity. A regression of P on H or on both H and X would yield no relationship between price and concentration, but it would be incorrect to conclude that an increase in C would not affect price. Once again, the issue here is not endogeneity, but that the estimated relationship between price and concentration does not inform the relationship of interest.

The third problem raised by Bresnahan is that less concentrated markets may involve more statistical cost draws and may therefore be comprised of lower cost firms on average. If it is not possible to control for all costs in equations like (1) and (2) (e.g., if marginal cost has a random component observed by the firm but not the analyst), then a positive relationship between price and concentration in the data may simply reflect higher costs in more concentrated markets rather than anticompetitive conduct. Although one could address this specific issue with data on costs, doing so would not address the fundamental issue raised in this paper—the absence of an economic interpretation of any statistical relationship between price and concentration, even controlling for other exogenous factors.

Bresnahan concluded his critique of price-concentration studies by observing that "[t]hese questions of interpretation are not unanswerable...The questions are, however, unanswered." (p. 1044). Nearly 20 years later when the next volume of the IO Handbook was published, the questions raised by Bresnahan remained unanswered. For example, Whinston (2007) is troubled by similar interpretation problems.

Whinston observes that equations like (1) are often interpreted as reduced forms, "[b]ut in fact, [equation (1)] is *not* a true reduced form." (p. 2412). This criticism flags the problem that the concentration measure is endogenous. In addition, Whinston notes that although it is possible to derive an estimating equation of a form like (1) under symmetric Cournot oligopoly with constant marginal cost and constant elasticity demand, when firms are asymmetric, "...a firm's supply relation is unlikely even to take a form like [equation (1)], in which rival's prices or quantities affect the firm's pricing only through a concentration measure like H. If so, [equation (1)] will be misspecified." (p. 2413). This observation is an implication of the points made in this paper. Summarizing the use of price-concentration methodology, Whinston concluded with the call for research quoted in the first paragraph above to determine whether the method yields answers that are "at least approximately correct."

Weiss and Wolak's (2007) Econometric Handbook chapter on structural modeling raises similar issues about profit-concentration studies of the 1960s and 70s. Their criticism can largely be understood by replacing price in conditions (1) and (2) with profit.[16] An early draft of their chapter (Reiss and Wolak, 2003) included a section headed "Putting the 'Econ' back into *Econ*ometrics."[17] A reasonable interpretation of their criticism is that equations like (1) and (2) can only be understood in the context of the economic model that generates them.

This paper makes two main contributions to this discussion. First, it boils down the forgoing criticisms into the core issue that makes price-concentration analysis problematic—the "invertibility problem." Second, it explains the implications of this problem for the interpretation of price-concentration studies in the literature. The most closely related papers are Farrell and Shapiro (1990a, 1990b), but this paper contrasts with their work in several ways. I focus on the relationship between price and concentration rather than the relationship between welfare and concentration as in their papers. The main reasons for this are that antitrust authorities are often concerned with the consumer welfare effects that arise from changes in price, and empirical researchers tend to focus on price rather than welfare because price data are readily available. A second difference is that my findings do not rely on the Cournot assumption. A third difference is that I examine a wider range of concentration measures, including the modified-HHI, which is the natural analog to the HHI for examining the effects of partial acquisitions of the type examined in Farrell and Shapiro (1990b). Finally, Farrell and Shapiro did not discuss the implications of their findings for using concentration as an explanatory variable in empirical analysis, which is the primary motivation for this paper.

Outline of the Remainder of the Paper. The remainder of the paper is organized as follows. Section II presents an oligopoly framework sufficiently general to accommodate the most theories of oligopoly interaction. The one novel aspect of the framework is the general treatment of mergers and partial ownership, which integrates the modified-HHI framework into the theory. Section III describes price-concentration analysis in this framework. Section IV formalizes and generalizes the impossibility result claimed in the introduction and overview. Section V discusses the implications for empirical work. Section VI concludes the paper.

II. Oligopoly with Free Entry and Partial Ownership

Assessing the economic implications of the relationship between price and concentration requires an economic model of the factors that determine these variables. A general model would include the following elements:

- a set of possible competitors;

[16]See in particular their Example 5, p. 4300.
[17]Reiss and Wolak (2003).

- entry relations that determine the set of active competitors and products;
- demand relations;
- relations that determine the ownership and control structure in the market that arises through mergers and acquisitions and the trading of stock by shareholders;
- supply relations that represent some underlying model of pricing;
- exogenous factors that shift the various relations.

Formally, such a model can be described by the following relationships:

$$P = f^P(Q, E, C, X^P) \quad \text{(Supply relations)} \tag{3}$$
$$Q = f^Q(P, E, C, X^Q) \quad \text{(Demand relations)} \tag{4}$$
$$E = f^E(P, Q, C, X^E) \quad \text{(Entry relations)} \tag{5}$$
$$C = f^C(P, Q, E, X^C) \quad \text{(Ownership-control relations).} \tag{6}$$

In this system, the Xs are vectors of exogenous variables, and the other variables are potentially endogenously determined. The variables P and Q are vectors of prices and quantities, respectively, and f^P and f^Q are the corresponding (vector-valued) functions that represent the supply and demand relations. The variable E is an "Entry" indicator vector that describes which firms and products are active. The function f^E represents an entry process, which is a mapping from the other variables into firms's choices about whether to be active and if so which products to sell. The variable C is an ownership-control variable summarizing the ownership and control of each firm (as described below). The function f^C allows for the possibility that C may be endogenous, although acquisitions that affect C are typically treated as exogenous in the literature. In econometric applications, these equations would also contain econometric errors and parameters to be estimated, both of which are omitted here for brevity.

A. The Ownership-Control Matrix

I model ownership and control using the partial ownership framework of O'Brien and Salop (2000). This framework is general enough to accommodate most changes in financial ownership that affect managerial incentives, including: small asset acquisitions (Farrel and Shapiro, 1990b), joint ventures (Reynolds and Snapp, 1986; Bresnahan and Salop, 1986), mergers that create or arise from situations of partial ownership (e.g., Besen et al., 1996), and partial acquisitions by institutional investors (Azar, Schmalz, and Tecu, 2017; Azar, Raina, Schmalze, 2016). This framework also provides a basis for generalizing the most widely-employed concentration measure, the HHI, to situations of partial ownership. The remainder of this subsection explains this framework in sufficient detail to make the main points in this paper.[18]

[18]The reader is also referred to O'Brien and Salop (2000), particularly Appendix C.

Let N^F be the number of active firms. Firm j's profit is $\pi_j(y)$ where $y = (y_1..., y_{N^F})$ is a vector of the firms' strategic choice variables (e.g., quantities, prices, or investment). Assume that π_j is continuously differentiable in all of its arguments. The firms are owned by N^O different owners who may have financial interests in multiple firms in the industry. Let β_{ij} be owner i's fractional ownership of firm j, such that $\sum_i \beta_{ij} = 1$ for all j.[19]

The question arises as to the objective function of firm j's manager in choosing y_j given potentially disparate incentives of the firm's owners.[20] Presumably, the manager takes into account the preferences of the owners. O'Brien and Salop (2000) capture this idea by assuming that the manager of each firm maximizes a weighted sum of the investment returns of its owners, where the weights reflect the degree of "control" or "influence" the owners have over the managers. Specifically, let γ_{ij} represent the weight the manager of firm j applies to the investment returns of owner i, with $\sum_i \gamma_{ij} = 1$. One can interpret γ_{ij} as the control that owner i has over the manager of firm j. For example, if $\gamma_{ij} = 1$, then firm j maximizes the investment returns of owner i. This is as expected if owner i has complete control over firm j.[21] If $\gamma_{ij} = 0$, then owner i has no control over the manager of firm j. This might arise, for example, if owner i's shares are are non-voting, or if its ownership share is too small to influence the manager. Values of γ_{ij} between zero and one reflect intermediate cases.[22] Complete mergers arise in this framework as special cases in which the ownership and control weights are such that the managers of the acquiring and acquired firms' each maximize the sum of the profits of the merging firms. Examples are given below.

Analytically, the investment returns to owner i are given by $\pi_i^{Own} = \sum_k \beta_{ik}\pi_k$. The profit objective of firm j's manager is therefore

$$\begin{aligned}
\pi_j^{Man} = \sum_i \gamma_{ij}\pi_i^{Own} &= \sum_i \gamma_{ij}\left[\beta_{ij}\pi_j + \sum_{k\neq j}\beta_{ik}\pi_k\right] \\
&= \Big(\sum_i \gamma_{ij}\beta_{ij}\Big)\pi_j + \sum_i\sum_{k\neq j}\gamma_{ij}\beta_{ik}\pi_k \\
&= \Big(\sum_i \gamma_{ij}\beta_{ij}\Big)\pi_j + \sum_{k\neq j}\Big(\sum_i \gamma_{ij}\beta_{ik}\Big)\pi_k \\
&\propto \pi_j + \sum_{k\neq j}\frac{\left(\sum_i \gamma_{ij}\beta_{ik}\right)}{\left(\sum_i \gamma_{ij}\beta_{ij}\right)}\pi_k \\
&= \pi_j + \sum_{k\neq j} C_{jk}\pi_k \qquad (7)
\end{aligned}$$

[19] Sums are taken over the entire relevant domain unless otherwise indicated.

[20] Canonical models in microeconomics skirt this issue by assuming that each manager's objective is the profit of the *firm* (e.g., the particular production unit or product(s) it manages). However, a complete analysis of mergers that accounts for partial ownership cannot avoid the issue.

[21] This assumption makes sense if owner i's consumption of the products produced in the industry is small relative to the owner's overall consumption, which is a standard assumption in partial equilibrium analysis.

[22] Solution concepts from cooperative game theory such as the Shapley Value, the Shapley-Shubik Power Index, and the Banzhaf Power Index might provide theoretical underpinnings for values of γ_{ij} between zero and 1. See, e.g., Azar (2016).

where $C_{jk} = (\sum_i \gamma_{ij}\beta_{ik})/(\sum_i \gamma_{ij}\beta_{ij})$. The first order condition for choosing y_j to maximize (7) is

$$\frac{\partial \pi_j}{\partial y_j} + \sum_{k \neq j} C_{jk} \frac{\partial \pi_k}{\partial y_j} = 0, \quad j = 1, ..., N^F. \tag{8}$$

The conditions in (8) underly the supply relation (3). This shows that acquisitions affect incentives through components of the *ownership-control matrix* $C = \left[C_{jk}\right]$, which captures the effects of common ownership. The first term in (8), $\partial\pi_j/\partial y_j$, is the derivative of manager j's objective ignoring any financial interests in rival firms held by owners that have any degree of control over firm j. The additional terms reflect how financial interests by firm j's owners in each other firm $k \neq j$ affect firm j's incentives through the $N \times (N-1)$ variables C_{jk}, $k \neq j$.[23]

To model a merger, joint venture, or a set of partial acquisitions, it is necessary to specify how ownership translates into control. Write $\gamma_{ij}(\beta)$ as the control owner i has over firm j given the ownership matrix $\beta = \left[\beta_{ij}\right]$. Natural assumptions are: zero ownership by owner i in firm j gives the owner no control; 100 percent ownership gives the owner complete control; the owner's control weight increases from 0 to 1 as its ownership fraction goes from 0 to 1. That is,

Assumption 1 $\quad \gamma_{ij}(\beta)\Big|_{\beta_{ij}=0} = 0, \quad \gamma_{ij}(\beta)\Big|_{\beta_{ij}=1} = 1, \quad \frac{\partial \gamma_{ij}(\beta)}{\partial \beta_{ij}} \geq 0.$

Further assumptions about the functions $\gamma_{ij}(\beta)$ determine the rate at which increases in ownership translate into control. For example, if owner i's share of firm j is a silent financial interest over some range $[\underline{\beta}_{ij}, \bar{\beta}_{ij}]$, then $\gamma_{ij}(\beta) = 0$ for all β_{ij} in this range.[24] Under the assumption that control is equal to the ownership interest (the "proportional control" scenario employed in empirical studies of common ownership by Azar, Schmalz, and Tecu, 2017 and Azar, Raina, and Schmalz, 2016), $\gamma_{ij}(\beta) = \beta_{ij}$.

A complete merger between firms j and k in an environment with no other partial ownership arises as a change in the ownership and control matrix from all zeros to $C_{jk} = C_{kj} = 1$ for j and k and all zeros for all other entries. This can arise in different ways in this framework depending on the nature of pre-merger ownership.

For example, suppose that each firm is initially a sole proprietor and that there is no pre-merger cross ownership. Suppose that the owner of firm 1 then acquires the fraction α of firm 2's stock. Write $\beta(\alpha)$ as the market ownership structure given α. In this scenario, the elements of the ownership-control matrix are

$$C_{jk}(\alpha) = \frac{\sum_i \gamma_{ij}\big(\beta(\alpha)\big)\beta_{ik}(\alpha)}{\sum_i \gamma_{ij}\big(\beta(\alpha)\big)\beta_{ij}(\alpha)}, \quad i = 1, ...N^O; \quad j,k = 1, ..., N^F \tag{9}$$

where the elements of the ownership matrix are $\beta_{11}(\alpha) = 1$, $\beta_{12}(\alpha) = \alpha$, $\beta_{21}(\alpha) = 0$, and $\beta_{22}(\alpha) = 1 - \alpha$. Using Assumption 1, straightforward calculations confirm that $C_{jk}(0) = 0$ for all $j \neq k$.

[23] It can be shown that an increase in C_{jk} raises the equilibrium price(s) if quantities are strategic substitutes under Cournot oligopoly, prices are strategic complements under Bertrand oligopoly, and a stability condition is satisfied.

[24] See Bresnahan and Salop (1986) and O'Brien and Salop (2000) for a practical discussion of different control scenarios.

Referencing (7), this means that the pre-merger objective of each manager j is to maximize π_j. Straightforward calculations also show that $C_{12}(1) = C_{21}(1) = 1$, and $C_{jk}(1) = 0$ for all other jk. Referencing (7), this means that the post-merger objectives of the managers controlling y_1 and y_2 are to maximize their joint profits $\pi_1 + \pi_2$, while the post-merger objective of each other manager j is to choose y_j to maximize π_j. The comparative statics from a small acquisition can be found in the standard way, (differentiating the system (3) through (6) with respect to α), and a complete merger occurs when α changes from 0 to 1.

If firms 1 and 2 have multiple owners prior to the merger, let α be the share of firm 2 transferred to the owners of firm 1, and assume that the transferred shares are allocated to firm 1's owners in proportion to their firm 1 shares. Then we can again express the ownership structure as $\beta(\alpha)$ and the acquisition matrix as $C(\alpha) = \left[C_{jk}(\alpha)\right]$, and a complete merger between firms 1 and 2 occurs when α changes from 0 to 1.

A third scenario for a complete merger is when a third party acquires firms 1 and 2. Let α_j be the third party's share of firm j ($j = 1, 2$), and write $\beta(\alpha_1, \alpha_2)$ as the ownership structure, so that the components of the acquisition matrix can be written $C_{jk}(\alpha_1, \alpha_2)$. A complete merger occurs when (α_1, α_2) changes from $(0, 0)$ to $(1, 1)$.

All of these pure merger scenarios assume that the only cross ownership that exists arises from stock acquisitions that reflect the merger. When pre-existing cross-ownership is important, this is readily accounted for in the appropriate pre-transaction β, or, in the context of the parameterization $\beta(\alpha)$, by using the appropriate pre-transaction value of α.

B. Concentration in the Theory of Partial Ownership

A concentration measure that generalizes the Herfindahl-Hirschman index (HHI) emerges from the preceding analysis under the assumption that firm are Cournot players. Under Cournot competition, the HHI is the share-weighted sum of the firms margins times the absolute value of the aggregate demand elasticity. Bresnahan and Salop (1986) defined the modified-Herfinadahl-Hircshman index in the same way for Cournot players involved in joint ventures or who take partial ownership positions in each other. O'Brien and Salop (2000) generalized this index to account for an arbitrary number of owners who may take ownership positions in multiple firms as in the theory presented above. They showed that the MHHI is

$$MHHI = \sum_j s_j^2 + \sum_{k \neq j} C_{jk} s_j s_k \tag{10}$$

$$= HHI + MHHID \tag{11}$$

where s_j is firm j's market share, HHI$= \sum_j s_j^2$, and MHHID$= \sum_j \sum_{k \neq j} C_{jk} s_j s_k$ is referred to as the "MHHI delta."[25] The MHHID measures the component of concentration due to common ownership. If some owners hold more than one firm and exert control over at least one of them,

[25]See O'Brien and Salop (2000), Appendix C.

one or more of the C_{jk} terms are non-zero. If all owners hold shares only in one firm, the MHHID is zero, and the MHHI collapses to the HHI.

C. The "Estimating Equations" Implied by the Theory

The model described by (3) through (8) is a general description of a market with oligopoly, free entry[26] and arbitrary ownership structures.[27] Following the literature, I refer to the relations in (3) through (6) as *structural* equations.[28] Under regularity conditions that allow application of the implicit function theorem, one can express the equilibrium values of the endogenous variables (indicated with superscript '*') as functions of the exogenous variables. This yields the *reduced-form* equations

$$P^* = f^{PR}(X) \tag{12}$$

$$Q^* = f^{QR}(X) \tag{13}$$

$$E^* = f^{ER}(X) \tag{14}$$

$$C^* = f^{CR}(X). \tag{15}$$

where X is a vector that include all exogenous variables.

The structural equations (3) through (6) and reduced-form equations (12) through (15) are all candidates for estimation using suitable techniques. In addition, one can work with "semi-reduced" equations obtained by eliminating P and Q from (3) and (4). This yields

$$P = f^{PS}(E, C, X^P, X^Q) \quad \text{(Semi-reduced price equation)} \tag{16}$$

$$Q = f^{QS}(E, C, X^P, X^Q) \quad \text{(Semi-reduced quantity equation)}, \tag{17}$$

which are also candidates for estimation that have clear interpretations grounded in economic theory.[29]Most merger retrospectives, which typically study the *ex post* effect of mergers on price, amount to estimating a semi-reduced price equation of the form (16) using time series or panel data, treating variation in the ownership-control matrix C as exogeneous.[30]

[26]For example, special cases would include the entry models of Novshek (1980) and Mankiw and Whinston (1986).

[27]The scenario presented in the introduction is a special case in which entry is fixed and the acquisition variable is exogenous.

[28]The terms "structural" and "reduced-form" can generate religious wars in econometrics, so let me be clear about the meaning of these terms in this paper. A "structural equation" is any equation that contains more than one endogenous variable and other exogenous variables. In the model in (3) through (6), potential endogenous variables include P, Q, E, and C, and the explicitly exogenous variables are X^P, X^Q, X^E, and X^C and any of the other variables that are not endogenous due to the nature of the system. Specifically, any one of the potential endogenous variables could be exogenous in the system (3) through (6) if its equilibrium value does not depend on the explicitly exogenous variables.

[29]If E and C are treated as exogenous, the semi-reduced equations for price and quantity are reduced-forms by the definition in footnote 28.

[30]See Ashenfelter et al. (2014) for a survey merger retrospectives.

III. Price-Concentration Analysis

Concentration H is generally defined as some function of shares, which in turn depend on prices and quantities. Using the semi-reduced price and quantity equations (16) and (17), we can write H as

$$H = f^{HS}(E, C, X^P, X^Q) \quad \text{(Semi-reduced concentration equation).} \tag{18}$$

Observe that the relationship between price and concentration implied by (16) and (18) is a parametric one: as the explanatory variables change, price and concentration vary according to the functions f^{PS} and f^{HS}.[31]

To explore this relationship, I consider three concentration measures: the N-firm concentration ratio, the HHI, and the MHHI. Let s_j be the revenue share of firm j, and let $s_{(k)}$ be the share of the kth largest firm. The N-firm concentration ratio is $CR_N = \max \sum_{k=1}^{N} s_{(k)}$. The HHI and MHHI were defined above.

Instead of estimating (16), which is derived from economic theory, price-concentration studies normally estimate equations of one of the following forms:

$$P = g^1(E, H, X^P, X^Q) \quad \text{(PC1),} \tag{19}$$

$$P = g^2(H, X^P, X^Q) \quad \text{(PC2),} \tag{20}$$

$$P = g^3(E, C, H, X^P, X^Q) \quad \text{(PC3)} \tag{21}$$

where H is defined as CR_N, the HHI, or the MHHI. In PC1, H replaces the ownership-control matrix C in (16). In PC2, H replaces both the entry variable E and the ownership-control matrix C. In PC3, H is added as a variable in the semi-reduced price equation.

I am interested in three questions regarding the methodology associated with (19) through (21):

1. Does the economic theory embodied in (3) through (18) yield the functional relationship (19), or (20)? (The relationship (21) trivially exists, as H is an extraneous variable in the semi-reduced price equation (16), which is assumed to exist.)

2. Does the theoretical relationship between price and concentration in (3) through (18) provide information about the relationship between price and C?

3. What is the interpretation of empirical analysis based on equations (19) through (21) in light of the economic theory embodied in (3) through (18) and the answer to questions 1 and 2?

It is important to be clear about the meaning of the statement "theory yields" (or "does not yield") the functional relationship $P = g^1(E, H, X^P, X^Q)$. Obviously, this relationship exists over some domains. For example, $g^1(\cdot)$ trivially exists when the domains of f^{PS} and f^{HS} both consist of the same single point, or even when it consists of two points. However, one of the purported

[31] The main difference between the analysis here and the special case discussed in the introduction is the inclusion of the entry variable E and the potential endogeneity of E and C.

merits of price-concentration analysis is that it does not require *a priori* restrictions on the details of the oligopoly model. The spirit of the method is supposed to be that of "reduced-form analysis," where the estimating equation is viewed as an approximation to the equlibrium price-concentration relationship that emerges from some underlying structural relationships, whatever they might be.[32] For the *methodology* embodied in the price-concentration relationship to capture this spirit, the functional relationship between price and concentration defined by g^1 should exist for all feasible values of the variables in a broad set of oligopoly environments. Henceforth, I refer to this set as the *relevant economic domain*. For the purposes of this paper, this domain is defined as follows.

Definition 1 *The* relevant economic domain $\mathcal{D}$ *consists of all functions* f^P, f^Q, f^E, f^C *and all values of* Q, P, E, C, *and* X *such that equations (3) through (6) comprise a well-defined theory of Cournot, Bertrand, or dominant firm/competitive fringe oligopoly.*

Sometimes we need to talk about the domain of the variables over which functions in the relevant economic domain are defined. Define this sub-domain as $\mathcal{V}(\mathcal{D})$.

IV. The Impossibility Result

In this section I establish the following Theorem.

Theorem 1 *The functional relationships (19) and (20) do not exist over the domain* $\mathcal{D}$ *when concentration is defined as the N-firm concentration ratio, the HHI, or the MHHI.*

The remainder of this section proves this theorem. In order to do so, I often work with with some limited domain $\mathcal{D}' \subset \mathcal{D}$ and show that the functional relationships (19) and (20) do not exist over $\mathcal{D}'$. Because the limited domain is a subset of the relevant economic domain, establishing non-existence over the limited domain establishes non-existence over the relevant economic domain.

Although it is not necessary to do so to establish the theorem, it is informative to consider separately cases in which C has a single dimension (e.g., firm 1 acquires a share of firm 2) and multiple dimensions (e.g., multiple owners take financial positions in multiple competing firms).

A. Single dimensional ownership and control

Consider an economic environment (domain) $\mathcal{D}'$ that consists of N owners and N single-product firms that are Cournot or Bertrand competitors. Owner 1 holds 100 percent of firm 1 and the fraction α of each of the other firms. Owner $i = 2, ..., N$ holds the fraction $1 - \alpha$ of firm i. For simplicity, assume that $C_{1k} = \alpha$ for all $k = 2, ..., N$. This case occurs when owner 1's investment in firm k is a silent financial interest, i.e., when owner 1's control weights are zero for each financial interest in firms other than firm 1, which it controls. All other ownership-control variables equal

[32] Of course, an equation like $P = g^1(E, H, X^P, X^Q)$ is not a true reduced form if E and H are endogenous. The point is that the spirit of price-concentration analysis is to avoid specifying the details of the underlying structural model.

zero because the owners of the other firms do not hold shares in rival firms. Assume that the components of X include the vector of marginal costs, $v = (v_1, ..., v_N)$, and vector of capacities $K = (K_1, ..., K_N)$. All firms are active at all marginal costs such that their equilibrium quantities are positive (there are no fixed costs). In this environment, the semi-reduced price and concentration equations (16) and (18) can be written as $P = f^{PS}(\alpha, v, K)$ and $H = f^{HS}(\alpha, v, K)$, respectively, because α completely determines C and (v, K) completely determines E. Denote the dependence of firm k's share on the ownership-control parameter and cost parameters as $s_k(\alpha, v, K)$. Assume that regularity conditions are satisfied such that these functions are continuous and differentiable. Assume that firm 1's price strictly rises and its share strictly falls with α, a typical case.[33] Finally, assume that $s_1(\alpha, v, K)$ is decreasing in firm 1's marginal cost, increasing in each rival's marginal cost, and that values of the marginal costs and capacity exist that generate every firm 1 share between zero and one. In summary, the domain $\mathcal{D}'$ includes a wide range of oligopoly environments in which the vector of marginal costs and capacities determine which firms will be active and common ownership has a single dimension.

The following Lemma facilitates the proof of Theorem 1.

Lemma 1 *A functional relationship $P = g(H, v, K)$ does not exist for any $g(\cdot)$ over $\mathcal{V}(\mathcal{D}')$ if $H = f^{HS}(\alpha, v, K)$ is not monotonic (hence not invertible) in α over $\mathcal{V}(\mathcal{D}')$.*

Proof: Suppose that H is not monotonic in α over $\mathcal{V}(\mathcal{D}')$. By continuity, there exists some value of H, say H', that corresponds to at least two different values of α. Because firm 1's price is strictly increasing in α over $\mathcal{V}(\mathcal{D}')$, this implies that H' corresponds to at least two different prices, which implies that the functional relationship $P = g(H, v, K)$ does not exist over $\mathcal{V}(\mathcal{D}')$. □

I complete the proof of the theorem for the single dimensional ownership and control case by showing that $H = f^{HS}(\alpha, v, K)$ is not monotonic in α for some values of v and K under all three definitions of concentration.

Concentration Defined as the N-firm Concentration Ratio. Suppose there are two competitors with no capacity constraints. The relevant N-firm concentration measure is the 1-firm concentration ratio. Thus, $H = \max\{s_k(\alpha, v, K) \mid k = 1, 2\}$. The derivative of H is

$$\frac{\partial H}{\partial \alpha} = \begin{cases} \frac{\partial s_1(\alpha,v,K)}{\partial \alpha} & \text{if } s_1(\alpha, v, K) < 1/2, \\ -\frac{\partial s_1(\alpha,v,K)}{\partial \alpha} & \text{if } s_1(\alpha, v, K) > 1/2. \end{cases} \tag{22}$$

Observe that the derivative of H changes sign at values of (α, v, K) such that $s_1(\alpha, v, K) = 1/2$. These values exist within the domain $\mathcal{V}(\mathcal{D}')$, implying that H is not monotonic in α over $\mathcal{V}(\mathcal{D}')$.

Concentration Defined as the HHI. In the two firm case, concentration is now $H = s_1(\alpha, v, K)^2+$

[33] Under Cournot oligopoly, price rises and firm 1's share declines with α if the equilibrium is locally strictly stable. Under differentiated Bertrand oligopoly, firm 1's share may rise or fall with α. An example of a case in which firm 1's price rises and its share falls with α is the case of constant elasticity demand. See the Appendix for these results.

$(1 - s_1(\alpha, v, K))^2$. The derivative is

$$\frac{\partial H}{\partial \alpha} = 2\frac{\partial s_1(\alpha, v, K)}{\partial \alpha}\left[2s_1(\alpha, v, K) - 1\right] \tag{23}$$

As with the 1-firm concentration ratio, the derivative changes sign at values of (α, v, K) such that $s_1(\alpha, v, K) = 1/2$. Thus, H is not monotonic in α over the domain $\mathcal{D}'$.

Concentration Defined as the MHHI. We impose the following additional restrictions (i.e., further limiting the economic domain): demand is linear, firm 1 has no capacity constraint, and firms other than firm 1 comprise a competitive fringe of symmetric firms with binding capacity constraints. This is a model of a dominant firm facing a competitive fringe with inelastic supply.

Let K^F be the total supply of fringe firms 2 through N. The concentration measure is now $H = MHHI = HHI + MHHID$. The HHI component is

$$\begin{aligned} HHI &= s_1^2 + \sum_{k \neq 1} s_k^2 \\ &= s_1^2 + (N-1)\left(\frac{1-s_1}{N-1}\right)^2 \\ &= s_1^2 + \frac{(1-s_1)^2}{N-1}. \end{aligned} \tag{24}$$

The MHHID component is

$$\begin{aligned} MHHID &= \sum_{k \neq 1} \alpha s_1 s_k \\ &= (N-1)\alpha s_1 \frac{(1-s_1)}{N-1} \\ &= \alpha s_1 (1-s_1). \end{aligned} \tag{25}$$

The concentration measure is therefore

$$H = HHI + MHHID = s_1^2 + \frac{(1-s_1)^2}{N-1} + \alpha s_1(1-s_1) \tag{26}$$

The derivative with respect to α is

$$\frac{\partial H}{\partial \alpha} = \frac{\partial s_1}{\partial \alpha}\left(\left(2(1-\alpha) + \frac{1}{N-1}\right)s_1 - \frac{2}{N-1} + \alpha\right) + s_1(1-s_1) \tag{27}$$

I show that if N is sufficiently large, the derivative changes sign for some ownership level $\alpha' \in (0, 1)$, which means that H is not monotonic in α. It then follows from Lemma 1 that the function $g(H, v, K)$ does not exist over the domain $\mathcal{D}'$.

Let $q_1^*(\alpha, v, K^F)$ be firm 1's optimal quantity given fringe capacity K^F. Under linear demand

and constant marginal cost, straightforward calculations show that $\partial q_1^*/\partial\alpha = -K^F/2$.[34] Firm 1's equilibrium share is $s_1 = q_1^*/(q_1^* + K^F)$. Differentiating with respect to α shows that $\partial s_1/\partial\alpha = -(1/2)(1-s_1)^2$. Substituting this expression into (27) yields the following expressions for the derivative of H with respect to α at the endpoints $\alpha = 0$ and $\alpha = 1$.

$$\left.\frac{\partial H}{\partial \alpha}\right|_{\alpha=0} = -\frac{1}{2}(1-s_1)^2\left(\left(2+\frac{1}{N-1}\right)s_1 - \frac{2}{N-1}\right) + s_1(1-s_1) \tag{28}$$

$$\left.\frac{\partial H}{\partial \alpha}\right|_{\alpha=1} = -\frac{1}{2}(1-s_1)^2\left(\frac{s_1-2}{N-1}+1\right) + s_1(1-s_1) \tag{29}$$

As the number of fringe firms becomes large, these derivatives become

$$\lim_{N\to\infty}\left.\frac{\partial H}{\partial \alpha}\right|_{\alpha=0} = (1-s_1)s_1^2 > 0, \tag{30}$$

$$\lim_{N\to\infty}\left.\frac{\partial H}{\partial \alpha}\right|_{\alpha=1} = \frac{(1-s_1)(3s_1-1)}{2} \quad (< 0 \text{ if } s_1 < 1/3). \tag{31}$$

Thus, for N sufficiently large, $\partial H/\partial\alpha > 0$ at $\alpha = 0$, and if $s_1 < 1/3$, then $\partial H/\partial\alpha < 0$ at $\alpha = 1$. By continuity, it follows that the derivative changes sign for some α' between zero and one. Therefore, H is not monotonic in α, and the function $P = g(\alpha, v, K)$ does not exist over the domain $\mathcal{D}'$, and thus does not exist over the relevant economic domain $\mathcal{D}$. This proves Theorem 1. □

Parameterized example for the single-dimensional ownership and control case. To gain intuition, consider Figure 1, which plots the HHI, MHHI, MHHID, and price as functions of the common ownership variable for an example involving a dominant firm. In this example, inverse demand is $P = 100 - q_1 - K^F$, $K^F = 50$, $v = 0$, and N is large, so that the shares of the fringe firms are small.

As the owner of the dominant firm acquires a greater percentage of the fringe (as α rises), price raises because the dominant firm cuts output. However, the concentration measures do not always rise with the amount acquired. The HHI in this example is approximately the square of the dominant firm's share. As the owner of the dominant firm acquires a greater percentage of the fringe, the HHI falls because because the dominant firm raises price and reduces its share. Thus, an increase in the HHI is not associated with an increase in price in this example. Perhaps this is not surprising given that the acquisition involves a change in common ownership rather than a complete merger, and the HHI does not take into account common ownership.

What may be more surprising is that the MHHI need not rise with fraction of the fringe that the dominant firm acquires. The MHHI equals the HHI plus the MHHID, where the MHHID component takes into account common ownership. The MHHID in this example is $\alpha s_1(1-s_1)$ (where the dominant firm is firm 1). The direct effect of an increase in α on the MHHID is

[34] The dominant firm's profit is $\pi = (1-q_1-K^F-v)(q_1+\alpha K^F)$. The first order condition for the profit maximizing quantity is $1-2q_1-v-(1+\alpha)K^F = 0$. The profit-maximizing quantity is therefore $q^* = (1-v-(1+\alpha)K^F)/2$, and $\partial q_1^*/\partial\alpha = -K^F/2$.

positive, but an indirect effect arises through the change in firm 1's share, and this effect can be negative. In particular, if firm 1's share exceeds 1/2, then a reduction in firm 1's share reduces the share component $s_1(1 - s_1)$ of the MHHID. Thus, if the dominant firm's share exceeds 1/2, the indirect effect on the MHHID is negative. As α rises and firm 1's share falls, this indirect effect eventually dominates the direct effect and reduces the MHHID. Because the MHHI is the sum of the HHI and MHHID and the HHI declines with α for all acquisitions, the MHHI also falls with α above some threshold level, which in this example is $\hat{\alpha} = 1/3$.

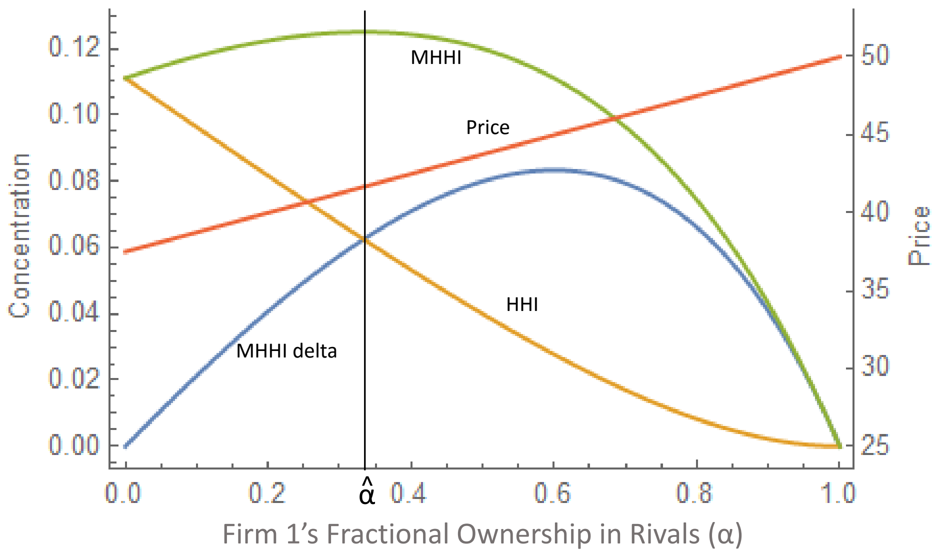

Figure 1: Relation between common ownership and price in a dominant firm example.

This example illustrates the general problem with using the relationship between price and concentration to infer the relationship between price and common ownership. Formally, because the MHHI is not monotonic in the acquisition α, the functional relationship between the equilibrium value of the MHHI and α is not invertible. There are two values of α that yield a given value of the MHHI. These two values of α generate two different prices. This means that the equilibrium price cannot be expressed as a function of the MHHI and the exogneous factors v and K^F. Practically, the relationship between price and the MHHI in data does not by itself provide information about how the acquisition affects price. In this example, an acquisition of any size unambiguously raises price, but the relationship between price and the MHHI induced by the acquisition may be positive or negative depending on the size of the pre- and post-acquisition shareholdings.

B. Multi-dimensional ownership and control

As noted earlier, it is not necessary to pursue the multi-dimensional case to establish Theorem 1, but it is instructive to do so because it illustrates the scope of the invertibility problem. For this case, consider a 2-firm Cournot oligopoly with no entry or exit and N^O owners who may take positions in both firms. The relevant off diagonals of the ownership-control matrix are C_{12}

and C_{21}. Omitting variables other than the ownership and control variables, the equilibrium price is $P^* = f^{PS}(C_{12}, C_{21})$. Write the equilibrium value of the MHHI as $H^* = f^{HS}(C_{12}, C_{21})$. The question is whether it is possible to write P^* as a function of H^* given the parametric equations that describe how the equilibrium values of these variables change with (C_{12}, C_{21}).

Fix H^* at $\bar{H}$, and define $\bar{C}_{12}(C_{21})$ as the value of C_{12} such that $\bar{H} = f^{HS}(\bar{C}_{12}(C_{21}), C_{21})$ (assume that regularity conditions are satisfied so that this function exists, at least near the values of (C_{12}, C_{21}) that yield $\bar{H}$). As C_{21} changes, C_{12} adjusts through $\bar{C}_{12}$ so that equilibrium concentration remains constant at $\bar{H}$.

Now observe that a necessary condition for the existence of a function $g(\cdot)$ such that $P^* = g(H^*)$ is that $f^{PS}(\bar{C}_{12}(C_{21}), C_{21})$ does not vary with C_{12}. The reason is that by construction, $f^{HS}(\bar{C}_{12}(C_{21}), C_{21})$ does not vary with C_{21}, so if $f(\bar{C}_{12}(C_{21}), C_{21})$ does vary with C_{21}, then there is more than one price associated with a given value of concentration, which means price is not a function of concentration. Mathematically, a necessary condition for the existence of a function mapping H^* to P^* is

$$\frac{dP^*}{dC_2} = \frac{\partial f^{PS}}{\partial C_{12}} \frac{\partial \bar{C}_{12}}{\partial C_{21}} + \frac{\partial f^{PS}}{\partial C_{21}} = \frac{\partial f^{PS}}{\partial C_{12}} \left(- \frac{\frac{\partial f^{HS}}{\partial C_{21}}}{\frac{\partial f^{HS}}{\partial C_{21}}} \right) + \frac{\partial f^{PS}}{\partial C_{12}} = 0$$

$$\implies - \frac{\frac{\partial f^{PS}}{\partial C_{12}}}{\frac{\partial f^{PS}}{\partial C_{21}}} = - \frac{\frac{\partial f^{HS}}{\partial C_{12}}}{\frac{\partial f^{HS}}{\partial C_{21}}} \tag{32}$$

$$\text{or} \quad MRS_P = MRS_H$$

where (32) uses the fact that $\partial \bar{C}_{12}/\partial C_{21} = -(\partial f^{HS}/\partial C_{21})/(\partial f^{HS}/\partial C_{12})$ given the definition of $\bar{C}_{12}$.

The ratios in (32) can be interpreted as marginal rates of substitution between C_{21} and C_{12} such that price and concentration remain constant. The left side of (32) describes how C_{12} changes with C_{21} to keep price constant (MRS_P), while the right side describes how C_{12} varies with C_{21} to keep concentration constant (MRS_H). Only if these marginal rates of substitution are equal is it possible to express the equilibrium price as a function of equilibrium concentration.

These marginal rates of substitution are equal under symmetry (both equal 1), but they generally are not equal otherwise. The following tractable example illustrates how condition (32) fails when firms have different costs.

Suppose there are two homogenous firms, one of which (firm 2) is strictly capacity-constrained, and one of which (firm 1) has constant marginal cost and can adjust quantity freely. In this example, a small increase in C_{21} (holding C_{12} fixed) does not affect firm 2's output decision because it is capacity constrained. That is, before the change in C_{21}, firm 2 would like to increase output but cannot, so after the change it will still want increase output and will not reduce it (if the change is small). Thus, equilibrium quantities and shares do not change from a small increase in C_{21}.

However, a change in C_{12} would affect output and price because it affects firm 1's optimal quantity choice.

Consider a small increase in C_{21}. Although this does not affect quantity and price (because firm 2's is capacity constrained), it does increase the MHHI, as the MHHID component is $MHHID = (C_{12} + C_{21})s_1 s_2$. In order to hold concentration constant, C_{12} must fall by the same amount that C_{21} rises, which means $MRS_H > 0$. Now consider MRS_P. Because the change in C_{21} does not change output, it will not change price. Therefore, the change in C_{12} that holds price constant is zero, i.e., $MRS_P = 0$, and thus condition (32) does not hold.

To see how this implies the nonexistence of a functional relationship between P^* and H^*, observe that a small increase in C_{21} offset by a reduction in C_{12} to hold the MHHI constant would generally reduce price, as the reduction in C_{12} causes firm 1 to increase its output. By continuity, there is a continuum of values $\left(\bar{C}_{12}(C_{21}), C_{21}\right)$ that yield the MHHI value $\bar{H}$, all with different values of $\bar{C}_{12}(C_{21})$ and therefore different prices. Thus, a continuum of prices is associated with each $\bar{H}$, which means it is not possible to express P^* as a function of H^*. □

Parameterized example for the multi-dimensional ownership and control case. The invertibility problem in the multi-dimensional case has a straightforward mathematical intuition. The oligopoly equilibrium generally depends on $N \times (N-1)$ terms in the common ownership incentive matrix $C = [C_{jk}]$ (the diagonals are irrelevant). In general, it is not possible to capture the effects of $N \times (N-1)$ incentive variables with a single-dimensional concentration index.

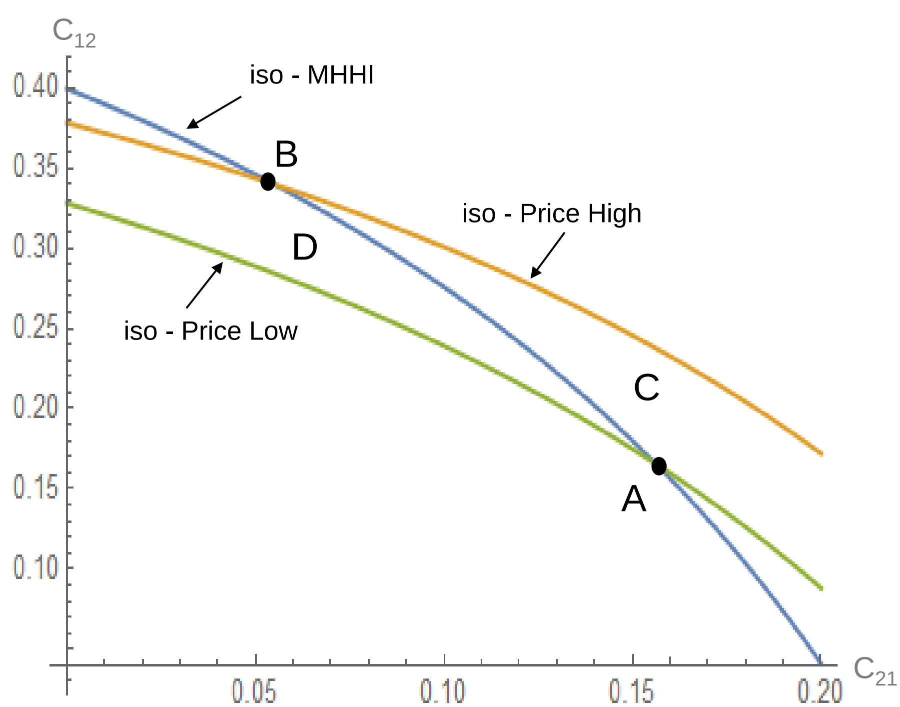

Figure 2: Relation between multi-dimensional common ownership and price.

The implications of this point are illustrated with the help of Figure 2, which considers a Cournot example with linear inverse demand $P = 1 - Q$ and two firms with marginal costs $v_1 = .1$ and $v_2 = .2$. The curve iso-MHHI represents pairs of common ownership variables C_{12} and C_{21} such that the MHHI is constant. The iso-Price curves represent pairs of common ownership variables such that the equilibrium price is constant at either a low price (iso-Price Low) or high price (iso-Price High). Observe that the iso-Price curves have a different slope than the iso-MHHI curve. This means that condition (32) is not satisfied, which is typically the case when competitors are asymmetric.

The common ownership pairs at points A and B yield the same MHHI, but different prices. Common ownership pairs in region C yield both a higher price and higher MHHI than common ownership at point A, but common ownership pairs in region D yields a higher price and *lower* MHHI than common ownership at point A. This means that variation in common ownership that raises price (variation from point A to points B, C, or D) may raise or lower the MHHI. Thus, the relationship between price and the MHHI by itself does not provide information about the relationship between price and common ownership.

C. The Cournot Relationship

A rigorous motivation for the MHHI arises in the Cournot model of oligopoly modified to take into account common ownership. It is instructive to consider why this relationship generally does not lead to a relationship expressing price as a function of the MHHI and exogenous variables.

Under Cournot behavior, the MHHI is proportional to the share-weighted sum of the margins:

$$H = \sum_{j=1}^{N} \frac{(P - v_j)}{P} s_j E \tag{33}$$

where E is the absolute value of the aggregate elasticity of demand. Taking differentials yields

$$\frac{\mathrm{d}P}{P} = \frac{\frac{d\bar{v}}{\bar{v}} + \sigma \frac{\mathrm{d}H}{H}}{1 - \sigma\mu} \tag{34}$$

where $\bar{v} = \sum_j s_j v_j$ is the share-weighted average marginal cost, $\mu = E'P/E$ is the elasticity of the demand elasticity, and $\sigma = H/(E - H) > 0$. For three commonly used demand curves—linear, log-log, and semi-log—it can be shown that the denominator on the right hand side of (34) is positive. In these cases, $\mathrm{d}P$ and $\mathrm{d}H$ have different signs if the change in the share-weighted average marginal cost ($\mathrm{d}\bar{v}$) and the change in the MHHI ($\mathrm{d}H$) have opposite signs and the cost change is larger in absolute value.

The parameterized examples presented in Sections IV.A. and IV.B illustrate this point. In single-dimensional case in IV.A, an increase in the dominant firm's ownership share of the fringe lowers the dominant firm's market share and reduces the MHHI if the dominant firm's ownership share is large enough. The reduction in the dominant firm's market share is accompanied by an increase in the share-weighted average marginal cost in the market. The reason this occurs is that the effective marginal cost of fringe firms equals the market price, while the dominant firm's marginal cost is lower. If the dominant firm's ownership share is large enough, the increase in average cost from the reduction in the dominant firm's share exceeds the reduction in the MHHI, as required in (34) for the acquisition to raise price.

In the multi-dimensional example in IV.B, a change in common ownership that raises C_{21} but reduces C_{12} so as to hold the MHHI constant leads to a reduction in price. This occurs because the reduction in C_{12} causes a reduction in price and an increase in the dominant firm's market share, which reduces the weighted average marginal cost.

An illuminating special case occurs when the weighted average marginal cost does not change, i.e., when $d\bar{v} = 0$. In this case, the change in price and the MHHI have the same signs. Other than this special case, there is no assurance that the change in price and MHHI will have the same sign.

D. Complete Mergers

The analysis to this point focuses on the relationship between price and concentration for small changes in partial ownership. I now show by example that the price change and concentration change may have different signs even for complete mergers.

Consider a market for a homogenous product with aggregate demand $Q(P) = 1 - P$ and three competing firms. Firm 1 has zero marginal cost and no capacity constraints; firm 2 has constant marginal cost equal to 1/4 and no capacity constraints; and firm 3 has zero marginal cost and a capacity constraint of 1/4. Under Bertrand competition, firm 1 charges a price just under 1/4 and sells a quantity equal to the difference between market demand at the price 3/4 and firm 3's supply of 1/4. Thus, firm 1 sells 1/2, firm 2 sells zero, and firm 3 sells 1/4. The shares are 2/3 for firm 1, 0 for firm 2, and 1/3 for firm 3, and the HHI is 5/9 $[= (2/3)^2 + (1/3)^2]$. Now suppose firms 1 and 2 merge. This removes firm 2 as a competitive constraint and makes the merged firm a dominant firm facing fringe supply of 1/4. The merged firm's optimal strategy is to sell 3/8. Total output is 5/8, and price is 3/8. The post-merger shares are 3/5 for firm 1 and 2/5 for firm 3, and the HHI is 13/25 $[= (3/5)^2 + (2/5)^2]$. The merger raises price from 1/4 to 3/8 and reduces the HHI from 5/9 to 13/25.

This example (like those presented for partial acquisitions) is not meant to suggest that merger-induced changes in price and concentration will always or even often have opposite signs. The point is simply to show that the relationship between price and concentration is not a robust predictor of the price effects of a change in ownership and control.

V. Implications for Empirical Analysis

A. Price-concentration analysis

The main implication of Theorem 1 is that there is no economic foundation for price-concentration analysis as embodied equations (PC1) and (PC2), which have the form of the equations estimated in most of the price-concentration literature. The reason is the invertibility problem identified in the introduction: concentration does not map into a unique ownership and control matrix, and therefore does not map into a unique price. The function (PC3) does arise from economic reasoning, but the concentration measure is an extraneous variable in that equation.

Why does this matter? Predictions in economics arise from comparative statics. However, the relationships (PC1) and (PC2) do not yield comparative statics predictions that inform the relationship of interest, which is this case is the relationship between price and common ownership (including merger).

The examples in Figures 1 and 2 showed that the sign of the price-concentration relation may differ from the sign of the relationship between price and a change in ownership. An additional problem is that price-concentration analysis may detect a price effect (in one direction or the other) in cases where the ownership change actually has no effect on pricing behavior. This is illustrated with the help of Figure 3, which shows how the MHHI, HHI, and MHHI delta vary with firm 1's share in a three firm oligopoly example that assumes, for simplicity, that firms 2 and 3 are symmetric. We can use this the figure to illustrate how price-concentration regressions can yield false positives about the competitive effects of common ownership.

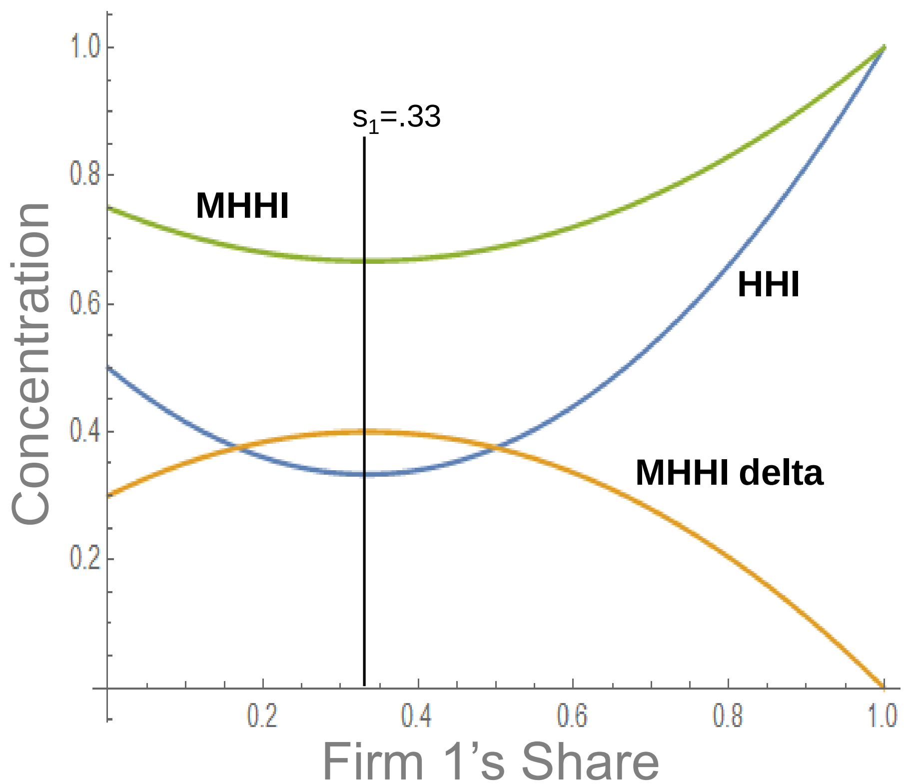

Figure 3: False positives and negatives in data.

As a first example, suppose that in calculating the MHHI, the researcher assumes that common ownership carries proportional control (control weights equal ownership interests), while in reality common owners have no influence over manager. Proportional control is the baseline assumption Azar et al.'s (2017) study of common ownership in airlines and Azar et al.'s (2016) study of by Azar et al. Imagine that a demand shock occurs that increases firm 1's share and price. This could occur, for example, if firm 1 responds more flexibly than rival firms to an increase in demand. For simplicity, think of this one change–from a low demand state to a high demand state–as the data.

Figure 3 shows that whether the MHHI, HHI, and MHHI delta rise or fall with firm 1's share depends on whether its share is in a range greater than or less than 1/3. For example, if firm 1's share is less than 1/3 before and after the demand shock, then the MHHI delta would rise in response to the shock along with price. In this case, the correlation between price and the MHHI delta would falsely associate the ownership change with a price increase. On the other hand, if firm 1's share is less than 1/3 before and after the shock, then the MHHI delta would fall with price, and this correlation would falsely associate the ownership change with a price decrease. The direction of the error depends on the value of firm 1's share. All one really learns from the sign of the correlation between price and concentration in this example is whether firm 1's share is greater than or less than 1/3!

As another example, suppose the data involves a shock that reduces firm 1 marginal cost and price and increases its share. Once again, whether the concentration measures rise or fall with the price reduction depends on firm 1's share before and after then cost shock. The price-concentration relationship discerned from this shock is the opposite of that associated with the demand shock, but again the error in the inference go either direction. The point is that the relationship between price and concentration by itself provides no information about the effect of ownership changes on price.

The false positives in these examples arise in part from endogeneity – i.e., the concentration measures are correlated with factors unobserved by the researcher that affect price. One might think that a systems estimation technique such as instrumental variables could address the problem, but the examples in Figures 1 and 2 show that this is not the case. If the price-concentration equation takes the forms (19) or (20), two-stage least squares (for example) would provide consistent estimates of coefficients that have no clear meaning. If the price-concentration equation takes the form of (21), the concentration measure is an extraneous variable in an otherwise properly specified reduced form. The coefficient on concentration should be zero, and if it is statistically different from zero, the implication is misspecification that need not have anything to do with the effects of common ownership.

B. Moving forward

Modern theories of oligopoly are described by simultaneous equations like those in (3) through (6). Inference in such environments involve estimating these equations using appropriate techniques, or estimating "reduced" versions derived by eliminating one or more endogenous variables. The system derived by eliminating *all* the endogenous variables and expressing each such variable as a function of the exogenous variables – commonly called the reduced form – is another way to go. None of these approaches involves estimating price as a function of concentration.[35]

The impact of mergers and common ownership arises through the common ownership variables C_{jk} that appear in the first order conditions (8). As an example, suppose that the only common ownership in a market involves firms 1 and 2 and that the firms are differentiated Bertrand competitors. Firm 1's first order condition for optimal pricing is then

$$p_1 = \underbrace{v_1 + \frac{-D_1}{\partial D_1/\partial p_1}}_{\text{Pre-merger FOC}} + \underbrace{C_{12}(p_2 - v_2)\delta_{12}}_{\text{Upward pricing pressure}} \tag{35}$$

where p_j is firm j's price, v_j is its marginal cost, D_1 is the demand for product 1, and $\delta_{12} = (-\partial D_2/\partial p_1)/(\partial D_1/\partial p_1)$ is the diversion ratio from product 1 to product 2. The second term on the right side of the equation is the upward pricing pressure due to common ownership,[36] which one can interpret as an opportunity cost to firm 1 of expanding quantity due to the impact on firm 2's profit. In a complete merger between firms 1 and 2, $C_{12} = 1$, and last term is the upward pricing pressure due to merger, where C_{12} changes from 0 to 1.[37] Of course, equilibrium effects are found by solving all of the first order conditions.[38]

Depending on the context, the values of the C_{jk} terms may or may not be known. For a complete merger between firms 1 and 2 in a market that involves no pre- or post-merger partial ownership, $C_{12} = C_{21} = 0$ before the merger and $C_{12} = C_{21} = 1$ after the merger. Given estimates of demand and cost conditions, one can use this knowledge to predict the effects of merger. Alternatively, in data that spans pre- and post-merger periods, one can exploit this variation in the data to help estimate cost and demand parameters.

In the case of partial ownership, the C_{jk} terms depend on the ownership matrix, which is known (or knowable), and the control weights, which are unknown.[39] There currently is no accepted theory of corporate control determining the allocation of influence across owners that have divergent interests. In the empirical work to date, the question has been whether minority shareholdings that involve common ownership confer a degree of control to the owners.

A general specification of how ownership maps into control is $\gamma = f^{\gamma}(\beta)$ where β is the ownership matrix and γ is the control matrix. However, this a function of high dimension and would be difficult to parameterize for empirical estimation. The approach taken in the empirical literature to date recognizes that the comparative statics effects of control flow through the C_{jk} terms (as illustrated in (35)), which subsume the ownership and control variables. For example, Azar et al. (2017)

[35]Concentration has an important role in industrial organization and in antitrust economics in particular, but that does not make it appropriate to put a share-based measure of concentration on the right hand side of a regression.

[36]See O'Brien and Salop (2000), Section V.B and Appendix C.3.

[37]See Farrell and Shapiro (2010).

[38]If a transaction causes a firm to stop selling one of the products, the entry equations (5) come into play as well.

[39]In particular, $C_12 = (\sum_i \gamma_{i1}\beta_{i2})/\sum_i \gamma_{i1}\beta_{i1}$ where $[\beta_{ij}]$ the matrix of ownership interest of each owner i in firm j and $[\gamma_{ij}]$ is the matrix of the owners' control weights.

estimate the following price regression to explain airline pricing:

$$p = X\theta + HHI\lambda_1 + MHHID\lambda_2 + \epsilon$$

where $MHHID = \sum_j \sum_{k \neq j} C_{jk} s_j s_k$ is calculated under a specific control assumption (proportional control, or control weights implied by the Banzhaff power index) and X is a vector of covariates. They interpret coefficient λ_2 as the effect of common ownership on price. Unfortunately, this approach suffers from the interpretation problems identified in this paper.[40]

Kennedy et al. (2017) conduct two analyses that address this problem. First, they estimate a price regression of the form

$$p = X\theta + h(C)\lambda + \epsilon$$

where $h(C)$ is an index that depends on the martix of the common ownership variables C (e.g., the mean, geometric mean, or inverse route distance-weighted mean of the C_{jk} terms by market).[41] This approach is consistent with economic theory in the sense that the explanatory variables are those predicted by theory. A weakness is that oligopoly theory generally predicts that the first order effects of common ownership on price depend on interactions between the common ownership matrix and other covariates. However, the high dimensionality of C makes it impractical to include all the interactions required to make the price regression flexible.

To address the inflexibility of the price regression, Kennedy et al. (2017) estimate a structural oligopoly model with supply equations (first order conditions) given by[42]

$$p_j = v_j + \frac{-D_j}{\partial D_j / \partial p_j} + \sum_{k \neq j} \tau C_{jk}(p_j - v_j)\delta_{jk}, \quad j = 1, ..., N^F. \tag{36}$$

Here, τ is a parameter to be estimated that, like the coefficients in the price regressions, nests the assumptions of zero control ($\tau = 0$) and the a particular control assumption used to calculate the matrix $[C_{jk}]$ (in which case $\tau = 1$). This method obviously relies on assumptions about the shape of the demand and cost functions, but given these assumptions, it captures the interactions among price, common ownership, and the other covariates.

Miller and Weinberg (2017) use a related procedure to test for price coordination between MillerCoors and Anheuser-Busch following the MillerCoors joint venture. Their method replaces the C_{jk} terms in (36) with '1' when the subscripts j and k reference two MillerCoors products or MillerCoors and Anheuser-Busch products. They interpret the estimate of τ as measuring the degree of price coordination between MillerCoors and Anheuser-Busch. One could conduct a similar test for the degree of price coordination among any subset of firms. The degree of coordination measured in this way is different than the degree of common ownership because the C_{jk} terms

[40] For a detailed review of Azar et al. (2017), see O'Brien and Waehrer, 2017.

[41] Gramlich and Grundl (2017) use a similar approach to examine the relationship between price and common ownership in banking.

[42] Kennedy et al assume demand takes a nested logit form, marginal cost is related to firm characteristics, and estimate the model using the generalized method of moments.

generally do not equal one. This suggests that it would be possible under Miller and Weinberg's interpretation to distinguish between the effects of common ownership and price coordination. However, there is no obvious reason why price coordination would have the property that the ratio of the cross terms in (36) equals the ratio of the value of diverted sales, $(p_j - v_j)\delta_{jk}/(p_l - v_l)\delta_{lm}$, as is true when τ is a singleton and $C_{jk} = 1$ for all pairs of coordinating firms. Because price coordination is typically understood as outcome of repeated interaction, a dynamic model is likely required to distinguish between the effects of common ownership and price coordination.

VI. Conclusion

This paper shows that oligopoly theory does not predict that equilibrium prices can be expressed as functions whose arguments consist of cost variables, demand variables, and share-based measures of concentration. Thus, the coefficients from price-concentration regressions have a dubious economic interpretation. This is true even if instrumental variables are used to deal with the endogeneity of concentration. The core issue is not econometric endogeneity, but the invertibility problem, i.e., the inability to map changes in the ownership and control structure into price or other variables of interest. In standard oligopoly models, a one-to-one mapping does not exist.

It remains somewhat fashionable in applied analysis to use the HHI and other concentration measures as explanatory variables in regressions. Such regressions may identify correlations in data, but they do not address questions about causal relationships, which are often the questions policy authorities need for their work.[43] It would be nice if it were possible to collapse the effects of events of concern into a single-dimensional concentration measure, but economic theory does not provide a basis for this practice.

When researchers estimate price-concentration relationships, they usually do so to answer a specific question. For example, the researcher may want to know whether one or more mergers in an industry or an increase in common ownership in an industry has led to higher prices, lower output, a change in wages, etc. One message of this paper is that regression analysis that uses concentration as an explanatory variable is an unreliable way to attempt to answer such questions. A second message is that tools exist to answer these questions properly.

The workhorse tool in economics is comparative statics. A merger, for example, is a particular change in common ownership, and researchers may be interested in determining the effects of this change on variables of interest. One can determine the effects of a merger, sequence of mergers, or other changes in common ownership on price or another variable of interest in two steps: by (i) estimating equations generated by an oligopoly model that takes into account the ownership and control structure; and (ii) conducting counterfactual analysis, which is the empirical implementation of comparative statics. Economic theory does not provide reasons to believe that price-concentration analysis is a good substitute for this type of analysis.

[43] For a detailed discussion of causal inference in science and econometrics, see Heckman (2005) and Heckman and Vytlacil (2007).

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